UNCERTAINTIES ON THE CP PHASE α
DUE TO PENGUIN DIAGRAMS

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Abstract

A major problem in the determination of the the CP angle α, that should be measured through modes of the type $B_d, \bar{B}_d \rightarrow \pi\pi, \cdots$, is the uncertainty coming from Penguin diagrams. We consider the different ground state modes $\pi\pi$, $\pi\rho$, $\rho\rho$, and, assuming the FSI phases to be negligible, we investigate the amount of uncertainty coming from Penguins that can be parametrized by a dilution factor D and an angle shift $\Delta\alpha$. The parameter D is either 1 or very close to 1 in all these modes, and it can be measured independently, up to a sign ambiguity, by the t dependence. Assuming factorization, we show that $\Delta\alpha$ is much smaller for the modes $\rho\pi$ and $\rho\rho$ than for $\pi\pi$, and we plot their allowed region as a function of α itself. Moreover, we show that most of the modes contribute to the asymmetry with the same sign, and define for their sum an effective D_{eff} and an effective $\Delta\alpha_{eff}$, an average of $\Delta\alpha$ for the different modes. It turns out that D_{eff} is of the order of 0.9, $\Delta\alpha_{eff}$ is between 5 % and 10 % and, relative to $\pi\pi$, the statistical gain for the sum is of about a factor 10. Finally, we compute the ratios $K\pi/\pi\pi, \cdots$ that test the strength of the Penguins and depend on the CP angles, as emphasized by Silva and Wolfenstein and by Deandrea et al.

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We will adopt Wolfenstein phase convention[1] and parametrization of the CKM matrix, with the expansion in powers of λ (up to order λ^3 included). In this convention all CKM matrix elements are real except V_{ub} and V_{td} and it is simple to identify which modes will contribute to the determination of the different angles on the unitarity triangle α, β and γ. In the Standard Model we have $|q/p| = 1$ to a very good approximation. In Wolfenstein phase convention $(q/p)_{B_d}$ is complex since it depends on V_{td} while $(q/p)_{B_s}$ is real as it depends on V_{ts}. For B decays, the CKM factor of the decay amplitudes is real for $b \to c$ transitions while it is complex for $b \to u$ transitions. This gives us four different possibilities according to the value of $Im\left[\frac{q}{p}\frac{\bar{M}}{M}\right]$:

1) $b \to u$ transitions of the B_d-$\bar{B}_d$ system, related to the angle α ;

2) $b \to c$ transitions of the B_d-$\bar{B}_d$ system, related to β ;

3) $b \to u$ transitions of the B_s-$\bar{B}_s$ system, related to γ ;

4) $b \to c$ transitions of the B_s-$\bar{B}_s$ system, related to the angle called β', vanishing at the considered order for the CKM matrix. Actually β' is of order λ^2.

Examples of the four types of modes, which are CP eigenstates, are respectively : $B_d, \bar{B}_d \to \pi^+\pi^-$; $B_d, \bar{B}_d \to \psi K_s, D^+D^-$; $B_s, \bar{B}_s \to \rho^0 K_s$, and $B_s, \bar{B}_s \to \psi\varphi$. Of course, this is only true in the tree approximation : *Penguin diagrams can complicate the picture and make uncertain the determination of the angles*, in some channels.

A systematic study of the contribution of Penguin diagrams to CP asymmetries has been done recently by A. Deandrea et al.[2], together with the study of the decay rates of modes where the Penguin diagrams can be dominating, as in CKM suppressed modes like $\bar{B}^0 \to K^-\pi^+$. Earlier literature on the importance of Penguin diagrams in B decays include the works by Gavela et al., Guberina and Peccei, Eilam[3], and recently, Deshpande and Trampetic and F. Buccella and collaborators[4]. On the other hand, Silva and Wolfenstein[5] and Deandrea et al.[2] have recently emphasized that the ratios of the type $K\pi/\pi\pi, \cdots$ give an independent determination of the CP angles, rather free of hadronic uncertainties if one assumes factorization. However, one should keep in mind that this

type of determination does not involve CP violation, and cannot make the economy of measuring CP asymmetries.

The aim of the present paper is to examine which are the most advantageous modes to determine the angle α of the unitarity triangle as far as the Penguin diagram uncertainty is concerned. This is the angle where the Penguin contribution can be the most important source of error. For the case of the angle β, although the modes of the type B_d, $\bar{B}_d \rightarrow D^+D^-, \cdots$ have also Penguin contributions, one has the very clean mode B_d, $\bar{B}_d \rightarrow \psi K_s$ which is, in practice, clean of such contamination. As far as the determination of the angle γ is concerned, one could in principle use the mode B_s, $\bar{B}_s \rightarrow \rho^0 K_s$. However, this is unlikely to be feasible since this mode is not only CKM suppressed (order λ^3 in amplitude), but also color suppressed, which means still another suppression factor of the order 0.2. Thus, in the case of the angle γ one should turn to modes of the type B_s, $\bar{B}_s \rightarrow K^-D_s^+, \cdots$ which are not CP eigenstates, and which are not affected by Penguin diagrams. The main hadronic uncertainties here concern the determination of the dilution factor D[6,7].

It has been pointed out by Gronau and London[8] that it could be possible to separate the Penguin contribution (pure $\Delta I = 1/2$) from the tree contribution (that has both $\Delta I = 1/2$ and $\Delta I = 3/2$ pieces) by isospin analysis of the different $\pi\pi$ channels. However, although this is in principle possible up to discrete ambiguities, it seems very difficult in practice, essentially because not only class I[9] decays like $\pi^+\pi^-$ are CKM suppressed, but class II decays like $\pi^0\pi^0$, color suppressed, have smaller branching ratios by about two orders of magnitude. Moreover, Deshpande and He[10] have recently pointed out that the Electroweak Penguins are not completely negligible in B decays (one operator has a sizeable Wilson coefficient), invalidating the isospin analysis since this contribution does not respect the usual isospin properties. In particular, not surprisingly, this new contribution enhances the relative weight of Penguins in the supressed modes like $\pi^0\pi^0$.

Therefore, it seems sensible to start with the dominant class I decays and investigate the uncertainty in the determination of the angle α coming from the Penguin contributions.

However, even here, there is still a further uncertainty coming from strong (FSI) phases. We shall neglect these possible FSI phases in this paper : on the one hand we do not know how to predict them, and furthermore, we can expect that, for a heavy system like the B with light decay products, they will be very small, since the final states will have large velocities. For sure, this point deserves further investigation.

We will restrict then to color allowed class I modes, since these will have the larger branching ratios, and moreover factorization is presumably for them on a rather firm ground. A point of warning must be made however. Since we are dealing with decays to light quarks, the heavy-to-light meson form factors at large momentum transfer will be involved, which are the worst known.

To summarize, we will consider the modes (with the different polarization states) :

$$B_d \ , \ \bar{B}_d \rightarrow \pi^+\pi^- \ , \ \pi^+\rho^- \ , \ \rho^+\pi^- \ , \ \rho^+\rho^- \quad . \tag{1}$$

Let us consider the final states :

$$|f> = \left|\pi^-(\mathbf{p})\pi^+(-\mathbf{p})\right\rangle$$
$$\left|\pi^-(\mathbf{p})\rho^+(\lambda=0,-\mathbf{p})\right\rangle$$
$$\left|\rho^-(\lambda=0,\mathbf{p})\pi^+(-\mathbf{p})\right\rangle$$
$$\left|\rho^-(\lambda=0,\mathbf{p})\rho^+(\lambda=0,-\mathbf{p})\right\rangle$$
$$\left|\rho^-(\lambda=\pm,\mathbf{p})\rho^+(\lambda=\pm,-\mathbf{p})\right\rangle \tag{2}$$

and their CP conjugate modes :

$$|\bar{f}> = \left|\pi^+(-\mathbf{p})\pi^-(\mathbf{p})\right\rangle$$
$$- \left|\pi^+(-\mathbf{p})\rho^-(\lambda=0,\mathbf{p})\right\rangle$$
$$- \left|\rho^+(\lambda=0,-\mathbf{p})\pi^-(\mathbf{p})\right\rangle$$
$$\left|\rho^+(\lambda=0,-\mathbf{p})\pi^-(\lambda=0,\mathbf{p})\right\rangle$$
$$\left|\rho^+(\lambda=\pm,-\mathbf{p})\rho^-(\lambda=\pm,\mathbf{p})\right\rangle \quad . \tag{3}$$

The spin quantization axis is along the line of flight of the decay products in the B_d rest frame.

The effective Hamiltonian, following Buras et al.[11] is given by[2,10] :

$$H = \frac{G}{\sqrt{2}} \left\{ V_{ud}^* V_{ub} \left(c_1 O_1 + c_2 O_2 \right) - V_{td}^* V_{tb} \left(c_3 O_3 + c_4 O_4 + c_5 O_5 + c_6 O_6 \right) \right\} + H_{EW}^{Penguin} \quad (4)$$

where the operators and Wilson coefficients are given by (at $\mu = m_b$) :

$$
\begin{aligned}
O_1 &= \left[\bar{u} \gamma_\mu \left(1 - \gamma_5 \right) b \right] \left[\bar{d} \gamma_\mu \left(1 - \gamma_5 \right) u \right] & c_1 &= 1.1502 \\
O_2 &= \left[\bar{u}_\alpha \gamma_\mu \left(1 - \gamma_5 \right) b_\beta \right] \left[\bar{d}_\beta \gamma_\mu \left(1 - \gamma_5 \right) u_\alpha \right] & c_2 &= -0.3125 \\
O_3 &= \left[\bar{d} \gamma_\mu \left(1 - \gamma_5 \right) b \right] \left[\bar{q} \gamma_\mu \left(1 - \gamma_5 \right) q \right] & c_3 &= 0.0174 \\
O_4 &= \left[\bar{d}_\alpha \gamma_\mu \left(1 - \gamma_5 \right) b_\beta \right] \left[\bar{q}_\beta \gamma_\mu \left(1 - \gamma_5 \right) q_\alpha \right] & c_4 &= -0.0373 \\
O_5 &= \left[\bar{d} \gamma_\mu \left(1 - \gamma_5 \right) b \right] \left[\bar{q} \gamma_\mu \left(1 + \gamma_5 \right) q \right] & c_5 &= 0.0104 \\
O_6 &= \left[\bar{d}_\alpha \gamma_\mu \left(1 - \gamma_5 \right) b_\beta \right] \left[\bar{q}_\beta \gamma_\mu \left(1 + \gamma_5 \right) q_\alpha \right] & c_6 &= -0.0459 \quad .
\end{aligned}
\tag{5}
$$

The Electroweak Penguin effective Hamiltonian $H_{EW}^{Penguin}$ has been recently computed by Deshpande and He, following Buras et al.[12]. Only one operator has a sizeable coefficient :

$$H_{EW}^{Penguin} = -\frac{G}{\sqrt{2}} V_{td}^* V_{tb} \left(c_7 O_7 + c_8 O_8 + c_9 O_9 + c_{10} O_{10} \right) \tag{6}$$

$$
\begin{aligned}
O_7 &= \tfrac{3}{2} \left[\bar{d} \gamma_\mu \left(1 - \gamma_5 \right) b \right] \left[e_q \, \bar{q} \gamma_\mu \left(1 + \gamma_5 \right) q \right] & c_7 &= -1.050 \times 10^{-5} \\
O_8 &= \tfrac{3}{2} \left[\bar{d}_\alpha \gamma_\mu \left(1 - \gamma_5 \right) b_\beta \right] \left[e_q \, \bar{q}_\beta \gamma_\mu \left(1 + \gamma_5 \right) q_\alpha \right] & c_8 &= 3.839 \times 10^{-4} \\
O_9 &= \tfrac{3}{2} \left[\bar{d} \gamma_\mu \left(1 - \gamma_5 \right) b \right] \left[e_q \, \bar{q} \gamma_\mu \left(1 - \gamma_5 \right) q \right] & c_9 &= -0.0101 \\
O_{10} &= \tfrac{3}{2} \left[\bar{d}_\alpha \gamma_\mu \left(1 - \gamma_5 \right) b_\beta \right] \left[e_q \, \bar{q}_\beta \gamma_\mu \left(1 - \gamma_5 \right) q_\alpha \right] & c_{10} &= 1.959 \times 10^{-3} \quad .
\end{aligned}
\tag{7}
$$

We will first make the calculation with the strong Penguin and then see that things are only very slightly modified by the electroweak one.

From the definitions

$$\langle P(p)\,|A_\mu|\,0\rangle = -if_P\,p_\mu$$

$$\langle V(p,\lambda)\,|V_\mu|\,0\rangle = m_V f_V \varepsilon_\mu^*(\lambda)$$

$$\langle P_i\,|V_\mu|\,P_j\rangle = \left(p_i^\mu + p_j^\mu - \frac{m_j^2 - m_i^2}{q^2}q_\mu\right)f_+(q^2) + \frac{m_j^2 - m_i^2}{q^2}q_\mu f_0(q^2)$$

$$\langle V_i\,|A_\mu|\,P_j\rangle = i\,(m_i + m_j)\,A_1(q^2)\left(\varepsilon_\mu^* - \frac{\varepsilon^* \cdot q}{q^2}q_\mu\right) -$$

$$- iA_2(q^2)\frac{\varepsilon^* \cdot q}{m_i + m_j}\left(p_i^\mu + p_j^\mu - \frac{m_j^2 - m_i^2}{q^2}q_\mu\right) + 2im_i A_0(q^2)\frac{\varepsilon^* \cdot q}{q^2}q_\mu$$

$$\langle V_i\,|V_\mu|\,P_j\rangle = \frac{2V(q^2)}{m_i + m_j}\varepsilon_{\mu\nu\rho\sigma}\,p_j^\nu\,p_i^\rho\,\varepsilon^{*\sigma} \tag{8}$$

we obtain, neglecting for the moment the Electroweak Penguin, the expressions :

$$M\left(\bar{B}_d^0 \to \pi^-(\mathbf{p})\pi^+(-\mathbf{p})\right) = \frac{G}{\sqrt{2}}if_\pi\left(m_B^2 - m_\pi^2\right)f_0^{ub}(m_\pi^2)$$

$$\times \left(V_{ud}^*V_{ub}a_1 - V_{td}^*V_{tb}\left\{a_4 + a_6\frac{2M_\pi^2}{(m_b - m_u)(m_u + m_d)}\right\}\right)$$

$$M\left(\bar{B}_d^0 \to \rho^-(\lambda = 0, \mathbf{p})\pi^+(-\mathbf{p})\right) = \frac{G}{\sqrt{2}}\,2f_\rho\,m_B\,f_+^{ub}(m_\rho^2)p$$

$$\times \left(V_{ud}^*V_{ub}\,a_1 - V_{td}^*V_{tb}\,a_4\right)$$

$$M\left(\bar{B}_d^0 \to \pi^-(\mathbf{p})\rho^+(\lambda = 0, -\mathbf{p})\right) = -\frac{G}{\sqrt{2}}\,2f_\pi\,m_B\,A_0^{ub}(m_\pi^2)p$$

$$\times \left(V_{ud}^*V_{ub}\,a_1 - V_{td}^*V_{tb}\left\{a_4 - a_6\frac{2M_\pi^2}{(m_b - m_u)(m_u + m_d)}\right\}\right)$$

$$M\left(\bar{B}_d^0 \to \rho^-(\lambda = 0, \mathbf{p})\rho^+(\lambda = 0, -\mathbf{p})\right) = i\frac{G}{\sqrt{2}}\,m_\rho\,f_\rho$$

$$\left[(m_B + m_\rho)\left(\frac{2p^2 + m_\rho^2}{m_\rho^2}\right)A_1^{ub}(m_\rho^2) - \frac{m_B^2}{m_B + m_\rho}\frac{2p^2}{m_\rho^2}A_2^{ub}(m_\rho^2)\right]$$

$$\times \left(V_{ud}^*V_{ub}\,a_1 - V_{td}^*V_{tb}\,a_4\right)$$

$$M^{pv}\left(\bar{B}_d^0 \to \rho^-(\lambda = \pm, \mathbf{p})\rho^+(\lambda = \pm, -\mathbf{p})\right) = i\frac{G}{\sqrt{2}}m_\rho f_\rho (m_B + m_\rho) A_1^{ub}(m_\rho^2)$$

$$\times (V_{ud}^* V_{ub}\, a_1 - V_{td}^* V_{tb}\, a_4)$$

$$M^{pc}\left(\bar{B}_d^0 \to \rho^-(\lambda = \pm, \mathbf{p})\rho^+(\lambda = \pm, -\mathbf{p})\right) = \pm i\frac{G}{\sqrt{2}}m_\rho f_\rho \frac{m_B}{m_B + m_\rho} 2V^{ub}(m_\rho^2)p$$

$$\times (V_{ud}^* V_{ub}\, a_1 - V_{td}^* V_{tb}\, a_4) \tag{9}$$

where we have used the notation of Deandrea et al.[2]

$$a_{2i-1} = c_{2i-1} + \frac{c_{2i}}{N_c} \qquad a_{2i} = c_{2i} + \frac{c_{2i-1}}{N_c} \qquad (i = 1, 2, 3) \quad . \tag{10}$$

As far as the combination of Wilson coefficients is concerned, these expressions agree with the ones given in the Tables of the paper by Deandrea et al.[2]. Let us make a comment on the Electroweak Penguins (6). For the modes under consideration the matrix elements of the operators O_7, O_8, O_9 and O_{10} are equal to those of respectively the operators O_5, O_6, O_3 and O_4. Therefore, the inclusion of the Electroweak Penguins simply makes the changes $a_4 \to a_4 + a_{10}$ and $a_6 \to a_6 + a_8$ in the preceding formulae, that amounts to changes only at the percent level in these coefficients. Notice that Deshpande and He find significant changes due to the Electroweak Penguins in the case of the $\pi^0\pi^0$, ... modes that are color suppressed and that we do not consider here.

Let us write the relevant asymmetries in our case for a definite type of final state like $|f\rangle = |\rho^-(\lambda = 0, \mathbf{p})\pi^+(-\mathbf{p})\rangle$ and its CP conjugate mode $|\bar{f}\rangle = -|\pi^-(\mathbf{p})\rho^+(\lambda = 0, -\mathbf{p})\rangle$.

The amplitudes will write, splitting into Tree and Penguin Amplitudes (remember that we neglect FSI phases) :

$$\bar{M}(f) = M_T\, e^{i\beta_T} + M_P\, e^{i\beta_P} \equiv M_1\, e^{i\beta_1}$$

$$\bar{M}(\bar{f}) = \bar{M}_T\, e^{i\beta_T} + \bar{M}_P\, e^{i\beta_P} \equiv M_2\, e^{i\beta_2}$$

$$M(\bar{f}) = -\left[M_T\, e^{-i\beta_T} + M_P\, e^{-i\beta_P}\right] \equiv -M_1\, e^{-i\beta_1}$$

$$M(f) = -\left[\bar{M}_T\, e^{-i\beta_T} + \bar{M}_P\, e^{-i\beta_P}\right] \equiv -M_2\, e^{-i\beta_2} \tag{11}$$

(M_1 and M_2 are the moduli of $\bar{M}(f)$ and of $\bar{M}(\bar{f})$).

Let us define $\Delta\alpha$ and n_f through :

$$\frac{q}{p} \frac{\bar{M}(f)}{M(f)} \cong -\frac{M_1}{M_2} e^{i(\varphi_M + \beta_1 + \beta_2)} \equiv -\eta_f \frac{M_1}{M_2} e^{2i(\alpha + \Delta\alpha)} \tag{12}$$

where α is the angle of the unitarity triangle, and η_f is a sign that depends on the final state. As we will see below, $\eta_f = +1$ for $\pi^+\pi^-$, $\rho^+\rho^-$ (parity violating part), $\pi^+\rho^- + \pi^-\rho^+$, and $\eta_f = -1$ for $\rho^+\rho^-$ (parity conserving part). The correction $\Delta\alpha$ ($\Delta\alpha \to 0$ for $M_P \to 0$) is given by :

$$\Delta\alpha = \frac{1}{2}(\beta_1 + \beta_2) - \beta_T \quad . \tag{13}$$

The time-dependent rates are proportional to :

$$R\left(B_{phys}^0(t) \to f\right) \sim [1 - R\cos(\Delta Mt) - D\sin[2(\alpha + \Delta\alpha)]\sin(\Delta Mt)]$$

$$R\left(\bar{B}_{phys}^0(t) \to f\right) \sim [1 + R\cos(\Delta Mt) + D\sin[2(\alpha + \Delta\alpha)]\sin(\Delta Mt)]$$

$$R\left(B_{phys}^0(t) \to \bar{f}\right) \sim [1 + R\cos(\Delta Mt) - D\sin[2(\alpha + \Delta\alpha)]\sin(\Delta Mt)]$$

$$R\left(\bar{B}_{phys}^0(t) \to \bar{f}\right) \sim [1 - R\cos(\Delta Mt) + D\sin[2(\alpha + \Delta\alpha)]\sin(\Delta Mt)] \tag{14}$$

where

$$R = \frac{(M_1)^2 - (M_2)^2}{(M_1)^2 + (M_2)^2} \qquad D = \frac{2M_1 M_2}{(M_1)^2 + (M_2)^2} \quad . \tag{15}$$

From these expressions, a useful CP asymmetry that we can consider :

$$\left\{R\left(B_{phys}^0(t) \to f\right) + R\left(B_{phys}^0(t) \to \bar{f}\right)\right\} - \left\{R\left(\bar{B}_{phys}^0(t) \to f\right) + R\left(\bar{B}_{phys}^0(t) \to \bar{f}\right)\right\} \sim$$

$$\sim D\sin[2(\alpha + \Delta\alpha)]\sin(\Delta Mt) \quad . \tag{16}$$

We can ask two interesting questions :

1) By how much does $\alpha + \Delta\alpha$ differ from α, the angle of the unitarity triangle ? $\Delta\alpha$ is a function of $\frac{V_{td}^*}{V_{ub}} = \frac{1 - \rho + i\eta}{\rho - i\eta}$.

2) By how much does D differ from 1 ? For the non-CP eigenstates D given by (15) depends also on (ρ, η). There are two kinds of uncertainties on D, then : i) the one

coming from the poor knowledge of the form factors involved in the calculation, and ii) the one coming from the Penguin contribution. However, at least in principle, one can have independent experimental information on D (up to a sign ambiguity since $|D| = \sqrt{1 - R^2}$) from the study of the time dependence through the $\cos(\Delta M t)$ in the formulae above.

Therefore, this second question is not as crucial as the first one, and the important uncertainty concerns $\Delta\alpha(\rho, \eta)$.

As we have shown in refs. 13 and 7, it is possible to consider the CP asymmetry for the whole sum $\pi^+\pi^- + \pi^+\rho^- + \rho^+\pi^- + \rho^+\rho^-$. Let us consider the sign of the asymmetry for the different contributions, relative to the one for $\pi^+\pi^-$ (parity violating, S wave, CP $= +$). In the case $\rho^+\rho^-$ (see the formulae (9)), we have the parity-violating piece contributing to both the longitudinal and transverse amplitudes (S and D waves, $CP = +$), with therefore the same sign as $\pi^+\pi^-$, while the parity-violating piece (P wave, $CP = -$) contributes with opposite sign. For the case $\pi^+\rho^- + \rho^+\pi^-$ (parity conserving, P wave), the situation is more involved. Let us consider only the spectator diagram, as we have done writting formulae (9). We can neglect the exchange diagram because it is color suppressed, vanishes in the chiral limit (current conservation if one assumes factorization), and moreover has further form factor suppressions, as discussed in ref. 13. We have shown in ref. 13 that in the heavy quark limit (c quark, as well as b quark), the process $B_d, \bar{B}_d \to (D^+D^{*-} + D^-D^{*+})(CP = +)$ is only allowed by the spectator diagram, while $B_d, \bar{B}_d \to (D^+D^{*-} + D^-D^{*+})(CP = -)$ is only allowed by the exchange diagram, which is very small. As emphasized in ref. 7, lattice calculations do not show changes in sign for the form factors and decay constants when extrapolating from heavy to light masses. Assuming that this is the case for all form factors involved, the processes $B_d, \bar{B}_d \to (\pi^+\rho^- + \rho^+\pi^-)(CP = +)$ and $B_d, \bar{B}_d \to (\pi^+\rho^- + \rho^+\pi^-)(CP = -)$ will be allowed by the spectator diagram, but with a cancellation between both contributions (π emission or ρ emission) in the latter, and a constructive interference in the former. Hence, $B_d, \bar{B}_d \to \pi^+\rho^- + \rho^+\pi^-$ will contribute to the asymmetry with the same sign as $\pi^+\pi^-$ although with a dilution

factor coming from the difference

$$\frac{\left[(A_\pi + A_\rho)^2 - (A_\pi - A_\rho)^2\right]}{\left[(A_\pi + A_\rho)^2 + (A_\pi - A_\rho)^2\right]} = \frac{2A_\pi A_\rho}{(A_\pi)^2 + (A_\rho)^2} \tag{17}$$

where A_π and A_ρ are the moduli of the amplitudes for π and ρ emission respectively.

For the whole sum[1], we will have, making a linear approximation on the corrections (see ref. 13) :

$$A(t) = D_{eff} \sin 2\left[\alpha + (\Delta\alpha)_{eff}\right] \sin \Delta M t \tag{18}$$

with

$$D_{eff} = \frac{2\sum_i \eta_i M_1^{(i)} M_2^{(i)} p_i}{\sum_i \left[\left(M_1^{(i)}\right)^2 + \left(M_2^{(i)}\right)^2\right] p_i} \qquad \Delta\alpha_{eff} \cong \sum_i D_i B_i (\Delta\alpha)_i \tag{19}$$

where the sum extends over all modes enumerated above with momenta p_i and branching ratios B_i. Of course, for the CP eigenmodes, $M_1 = M_2$. These amplitudes can be read from (9) and (11). Notice that only the P-wave parity conserving $\rho\rho$ mode contributes negatively to the numerator in this expression. In the expression of $\Delta\alpha_{eff}$, D_i are the individual dilution factors and R_i the corresponding branching ratios relative to the total sum of ground state modes, given in the Table 1. We obtain finally :

$$\Delta\alpha_{eff} = Arg\left[1 + 0.029\frac{1 - \rho + i\eta}{\rho - i\eta}\right] \tag{20}$$

essentially because the $\rho\rho$ modes dominate.

In Table I we summarize the results, giving the branching ratios for the different modes, and the dilution factors and corresponding $\Delta\alpha$. We use the value $\frac{2M_\pi^2}{(m_b - m_u)(m_u + m_d)} = 0.55$ for $m_u + m_d = 15$ MeV and $m_b = 4.7$ GeV,

$$\frac{V_{td}^* V_{tb}}{V_{ud}^* V_{ub}} = \frac{1 - \rho + i\eta}{\rho - i\eta} = -\left|\frac{1 - \rho + i\eta}{\rho - i\eta}\right| e^{-i\alpha} = -\left|\frac{V_{td}}{V_{ub}}\right| e^{-i\alpha} \quad . \tag{21}$$

Notice that $\Delta\alpha$ for individual modes is independent of the heavy-to-light form factors.

In Fig. 1 we plot the allowed region in the plane (ρ, η) taking into account present theoretical and experimental uncertainties[14] and the corresponding allowed domain in the plane $(\sin 2\alpha, \sin 2\beta)$. In Figs. 2 we plot $\Delta \alpha$ for the different modes and for their sum as a function of $\sin 2\alpha$ itself : $\sin 2\alpha$ and $\Delta \alpha$ as well are both functions of (ρ, η).

The rates have been computed by extracting the $B \rightarrow \pi(\rho)$ form factors from the data on the D semileptonic form factors $D \rightarrow K(K^*)$ at $q^2 = 0$[15] :

$$f_+^{sc}(0) = 0.77 \pm 0.04$$

$$V^{sc}(0) = 1.16 \pm 0.16$$

$$A_1^{sc}(0) = 0.61 \pm 0.05$$

$$A_2^{sc}(0) = 0.45 \pm 0.09 \quad . \tag{22}$$

We use the following prescriptions, motivated by our study of the data on an overall fit to these D semileptonic data and on $B \rightarrow \psi K(K^*)$[16] and our model of semileptonic heavy meson form factors[17] :

1) Let us begin by using exact SU(3) to relate $D \rightarrow K(K^*)$ to $D \rightarrow \pi(\rho)$ at $q^2 = 0$. Below we will see the sensitiveness to this hypothesis. It must be pointed out that SU(3) is just a simplifying assumption to get information on the $D \rightarrow \pi(\rho)$ form factors. Preliminary data on the ratio $D \rightarrow \pi/D \rightarrow K$ is consistent with SU(3) but also large SU(3) breaking is allowed by the present error.

2) We extrapolate the form factors $D \rightarrow \pi(\rho)$ from $q^2 = 0$ to $q^2 = q_{max}^2$ by a q^2 extrapolation, taking

$$\frac{f_0(q^2)}{f_0(0)} = \frac{A_1(q^2)}{A_1(0)} = 1$$

$$\frac{f_+(q^2)}{f_+(0)} = \frac{1}{\left[1 - \frac{q^2}{(M_D + M_\pi)^2}\right]}$$

$$\frac{A_0(q^2)}{A_0(0)} = \frac{A_2(q^2)}{A_2(0)} = \frac{V(q^2)}{V(0)} = \frac{1}{\left[1 - \frac{q^2}{(M_D + M_\rho)^2}\right]} \quad . \tag{23}$$

For the unmeasured form factors we take $f_0(0) = f_+(0)$ and $A_0(0) = A_1(0)$.

3) We extrapolate the $D \to \pi(\rho)$ form factors at their q^2_{max} to the $B \to \pi(\rho)$ form factors at their q^2_{max} by using the softened heavy-to-light scaling laws[15] :

$$\frac{f^{db}_+(q^2_{max})}{f^{dc}_+(q^2_{max})} = \left(\frac{M_D}{M_B}\right)^{\frac{1}{2}} \left(\frac{M_B + M_\pi}{M_D + M_\pi}\right)$$

$$\frac{f^{db}_0(q^2_{max})}{f^{dc}_0(q^2_{max})} = \left(\frac{M_B}{M_D}\right)^{\frac{1}{2}} \left(\frac{M_D + M_\pi}{M_B + M_\pi}\right)$$

$$\frac{A^{db}_0(q^2_{max})}{A^{dc}_0(q^2_{max})} = \frac{A^{db}_2(q^2_{max})}{A^{dc}_2(q^2_{max})} = \frac{V^{db}(q^2_{max})}{V^{dc}(q^2_{max})} = \left(\frac{M_D}{M_B}\right)^{\frac{1}{2}} \left(\frac{M_B + M_\rho}{M_D + M_\rho}\right)$$

$$\frac{A^{db}_1(q^2_{max})}{A^{dc}_1(q^2_{max})} = \left(\frac{M_B}{M_D}\right)^{\frac{1}{2}} \left(\frac{M_D + M_\rho}{M_B + M_\rho}\right) \tag{24}$$

4) Finally we extrapolate the $B \to \pi(\rho)$ form factors from q^2_{max} to $q^2 = M^2_\pi$ or M^2_ρ by using pole relations (23), except for the obvious replacement $m_D \to m_B$.

To test the stability of the results on our assumptions, we have made a number of changes. Starting from the central values (22) and extrapolating following the prescription described above[16], we have used exact SU(3) alternatively at $q^2 = q^2_{max}$ for $D \to K(K^*)$ or at $q^2 = 0$ for $B \to K(K^*)$ or at $q^2 = q^2_{max}$ for $B \to K(K^*)$. The results are rather stable. For example, the rate $B_d \to \pi^+\pi^-$ changes by 20 %, and D_{eff} for the whole sum by 6 % (it increases). If instead of the central values (22) we adopt the best fit to $D \to K(K^*)$ semileptonic form factors and $B \to \psi K(K^*)$ decay rates[16], we obtain results that are very close to the previous ones.

Finally, as a consistency test and to have an independent estimation of the magnitude of the Penguin diagrams, we will consider CP conserving processes, CKM suppressed modes where the Penguin can be dominant and which have the same topology as the modes interesting for CP violation discussed above. Their relative magnitude will be a precise test of the magnitude of the Penguins. The formulae for the amplitudes $M(\bar{B}^0_d \to K^-\pi^+)$, $M(\bar{B}^0_d \to K^{*-}\pi^+)$, $M(\bar{B}^0_d \to K^-\rho^+)$, $M(\bar{B}^0_d \to K^{*-}\rho^+)$, $M^{pv}(\bar{B}^0_d \to K^{*-}(\lambda = \pm)\rho^+(\lambda =$

$\pm$)), $M^{pc}(\bar{B}_d^0 \to K^{*-}(\lambda = \pm)\rho^+(\lambda = \pm))$ can be obtained from (9) by simply replacing V_{ud} and V_{td} by V_{us} and V_{ts}, and, when they are emitted, π and ρ by K and K^*.

We find, taking for all form factors $F^{ub}(m_K^2) = F^{ub}(m_\pi^2)$, $F^{ub}(m_{K^*}^2) = F^{ub}(m_\rho^2)$, but keeping SU(3) breaking for f_K/f_π, etc., and adopting the value $\frac{2M_K^2}{(m_b - m_u)(m_u + m_s)} = 0.67$ ($m_s = 150$ MeV) :

$$\frac{\Gamma\left(\bar{B}_d^0 \to \pi^+ K^-\right)}{\Gamma\left(\bar{B}_d^0 \to \pi^+ \pi^-\right)} = 0.07 \, \frac{|\rho - i\eta - 1.12|^2}{|\rho - i\eta + 0.05(1 - \rho - i\eta)|^2}$$

$$\frac{\Gamma\left(\bar{B}_d^0 \to \pi^+ K^{*-}\right)}{\Gamma\left(\bar{B}_d^0 \to \pi^+ \rho^-\right)} = 0.06 \, \frac{|\rho - i\eta - 0.59|^2}{|\rho - i\eta + 0.03(1 - \rho + i\eta)|^2}$$

$$\frac{\Gamma\left(\bar{B}_d^0 \to \rho^+ K^-\right)}{\Gamma\left(\bar{B}_d^0 \to \rho^+ \pi^-\right)} = 0.07 \, \frac{|\rho - i\eta - 0.05|^2}{|\rho - i\eta + 0.01(1 - \rho + i\eta)|^2}$$

$$\frac{\Gamma\left(\bar{B}_d^0 \to \rho^+ K^{*-}\right)}{\Gamma\left(\bar{B}_d^0 \to \rho^+ \rho^-\right)} = 0.06 \, \frac{|\rho - i\eta - 0.59|^2}{|\rho - i\eta + 0.03(1 - \rho + i\eta)|^2} \quad . \tag{25}$$

In Fig. 3 we plot these ratios as a function of $\sin 2\alpha$ and in Fig. 4 as a function of $\sin 2\beta$. We see that, even considering the present uncertainties on the coordinates (ρ, η), these rates are rather sensitive to the precise value of $\sin 2\beta$. Low values of these ratios could also give precious information on $\sin 2\alpha$. Needless to say, the measurement of these ratios would be a consistency test (modulo the factorization approximation) but cannot make the economy of measuring directly the CP angles through CP violating processes.

In conclusion, we have shown that, neglecting FSI phases, the uncertainties $\Delta \alpha$ coming from Penguin diagrams are smaller for the modes $\rho\pi$ and $\rho\rho$ than for $\pi\pi$. Moreover, summing over all these modes leads to an uncertainty $\Delta\alpha/\alpha$ of the order 5 to 10 %. The dilution factor is very close to 1 even for the whole sum and one wins an order of magnitude in statistics. However, one must keep in mind that these results are sensitive to the Ansatz for the heavy-to-light meson form factors. Our model relies on a combination of theoretical constraints and data on semileptonic D mesons decays (subject to corrections of the order $1/m_c$) and on non-leptonic B decays (here the serious uncertainty comes from the factorization hypothesis). Needless to say that the dependence of the form factors on masses and q^2 could be quite different from our expectations, even if the latter stems from

an extensive study[16]. Therefore, the present analysis must be considered as preliminary. Further knowledge on the many uncertainties involved (q^2 and mass dependence of heavy-to-light form factors, FSI phases, accuracy of factorization) should be included in future analyses along the same lines.

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Figure Captions

Fig. 1 Present allowed domain for the coordinates (ρ, η) and for $(\sin 2\alpha, \sin 2\beta)$. The theoretical and experimental uncertainties are specified.

Fig. 2 The uncertainty $\Delta\alpha$ as a function of α for the modes $\pi^+\pi^-$, $\pi^+\rho^- + \pi^-\rho^+$ and $\rho^+\rho^-$ and for the sum of all these modes.

Fig. 3 The ratios of rates $\pi^+K^-/\pi^+\pi^-$, $\pi^+K^{*-}/\pi^+\rho^-$, $K^-\rho^+/\pi^-\rho^+$ and $\rho^+K^{*-}/\rho^+\rho^-$ as a function of $\sin 2\alpha$.

Fig. 4 The ratios of rates $\pi^+K^-/\pi^+\pi^-$, $\pi^+K^{*-}/\pi^+\rho^-$, $K^-\rho^+/\pi^-\rho^+$ and $\rho^+K^{*-}/\rho^+\rho^-$ as a function of $\sin 2\beta$.

Table 1

Decay mode	BR	D	$\Delta\alpha$
$\bar{B}_d^0 \to \pi^+\pi^-$	1.94×10^{-5}	1	Arg $[1 + 0.055\frac{1-\rho+i\eta}{\rho-i\eta}]$
$\bar{B}_d^0 \to \pi^+\rho^-$	1.53×10^{-5}		
$\bar{B}_d^0 \to \rho^+\pi^-$	5.07×10^{-5}		
$\bar{B}_d^0 \to \pi^+\rho^- + \rho^+\pi^-$	6.60×10^{-5}	0.84	Arg $[1 + 0.019\frac{1-\rho+i\eta}{\rho-i\eta}]$
$\bar{B}_d^0 \to \rho^+\rho^-(L)$	1.26×10^{-4}	1	Arg $[1 + 0.029\frac{1-\rho+i\eta}{\rho-i\eta}]$
$\bar{B}_d^0 \to \rho^+\rho^-(T,pv)$	0.24×10^{-5}	1	Arg $[1 + 0.029\frac{1-\rho+i\eta}{\rho-i\eta}]$
$\bar{B}_d^0 \to \rho^+\rho^-(T,pc)$	0.48×10^{-5}	-1	Arg $[1 + 0.029\frac{1-\rho+i\eta}{\rho-i\eta}]$
$\bar{B}_d^0 \to \rho^+\rho^-(T)$	0.72×10^{-5}	-0.33	
$\bar{B}_d^0 \to \rho^+\rho^-$	1.37×10^{-4}	0.93	Arg $[1 + 0.029\frac{1-\rho+i\eta}{\rho-i\eta}]$
$\bar{B}_d^0 \to \pi^+\pi^- + \pi^+\rho^-$ $+\rho^+\pi^- + \rho^+\rho^-$	1.94×10^{-4}	0.91	Arg $[1 + 0.029\frac{1-\rho+i\eta}{\rho-i\eta}]$

The uncertainty $\Delta\alpha$ and the dilution factor D coming from Penguin diagrams and the branching ratios for the different modes (L and T denote longitudinal and transverse ; pc and pv mean parity conserving and parity violating).
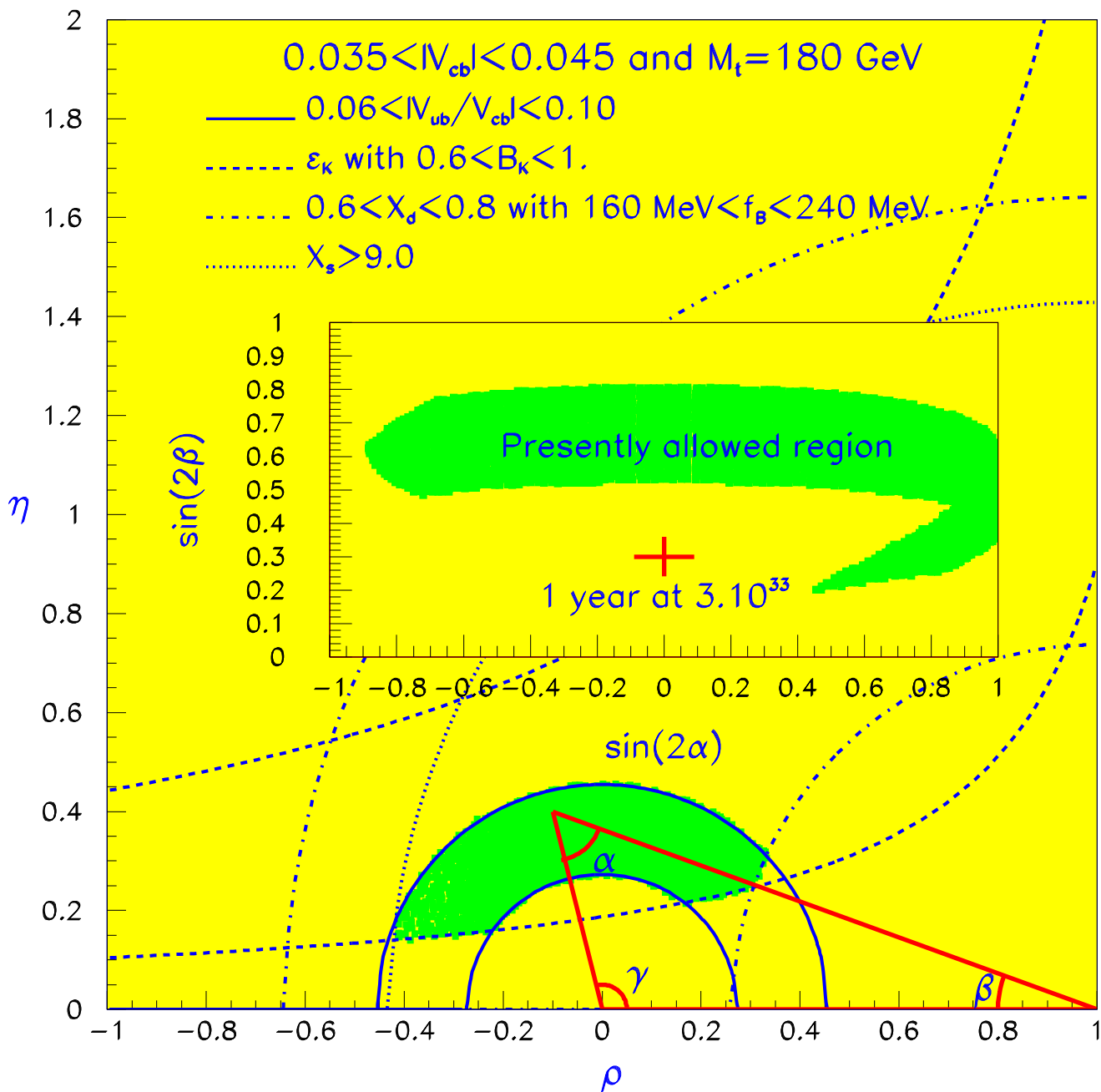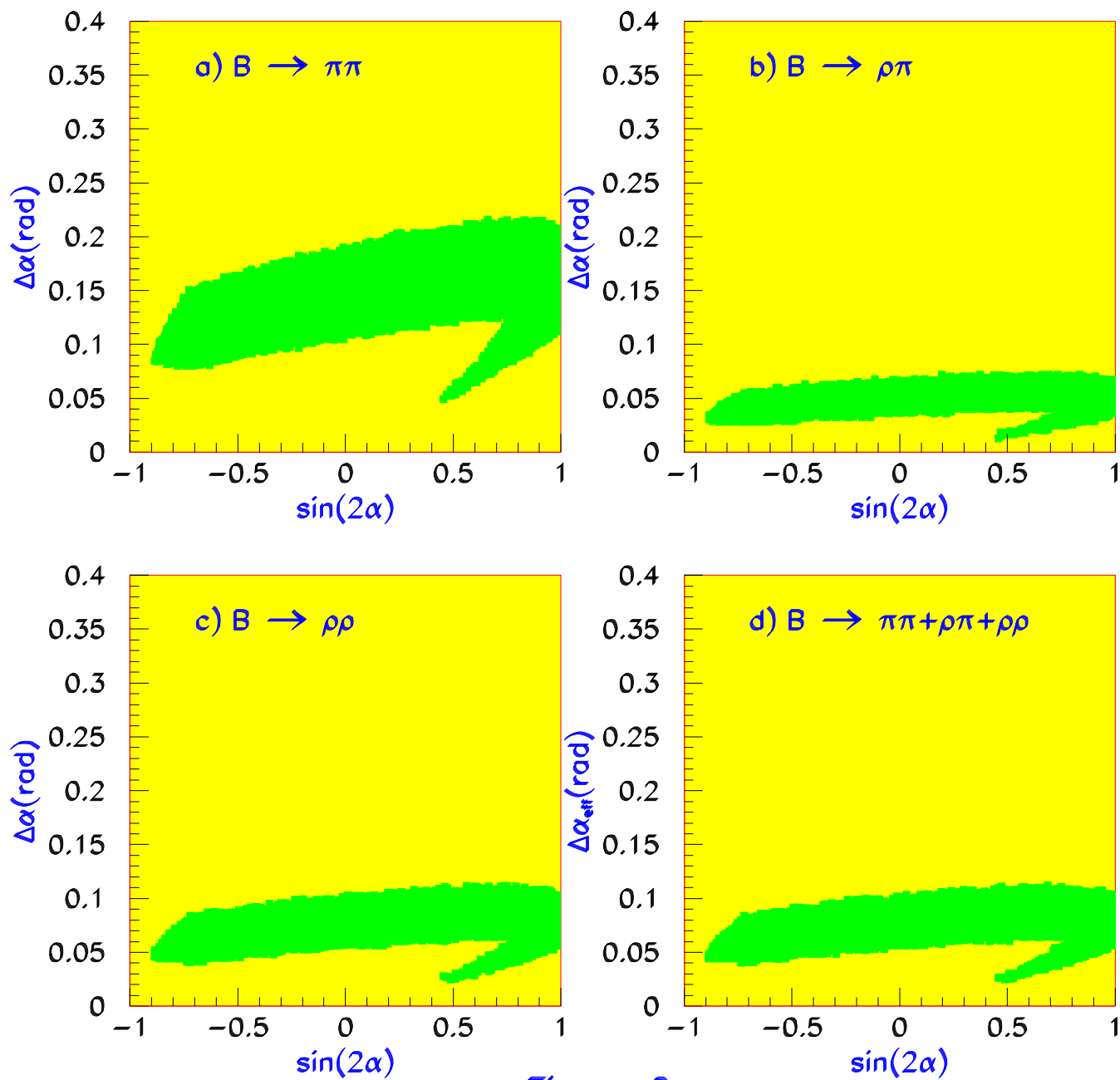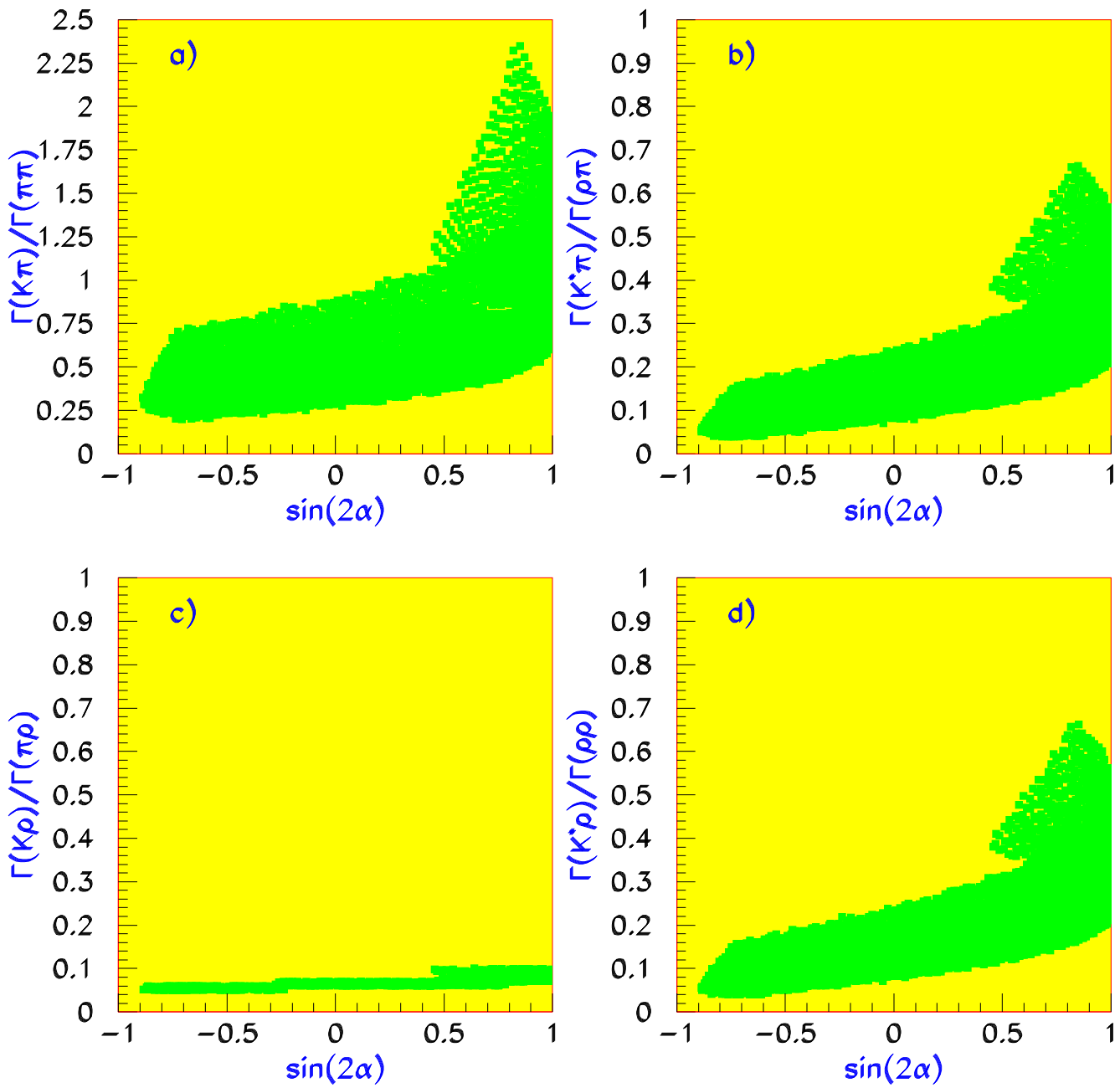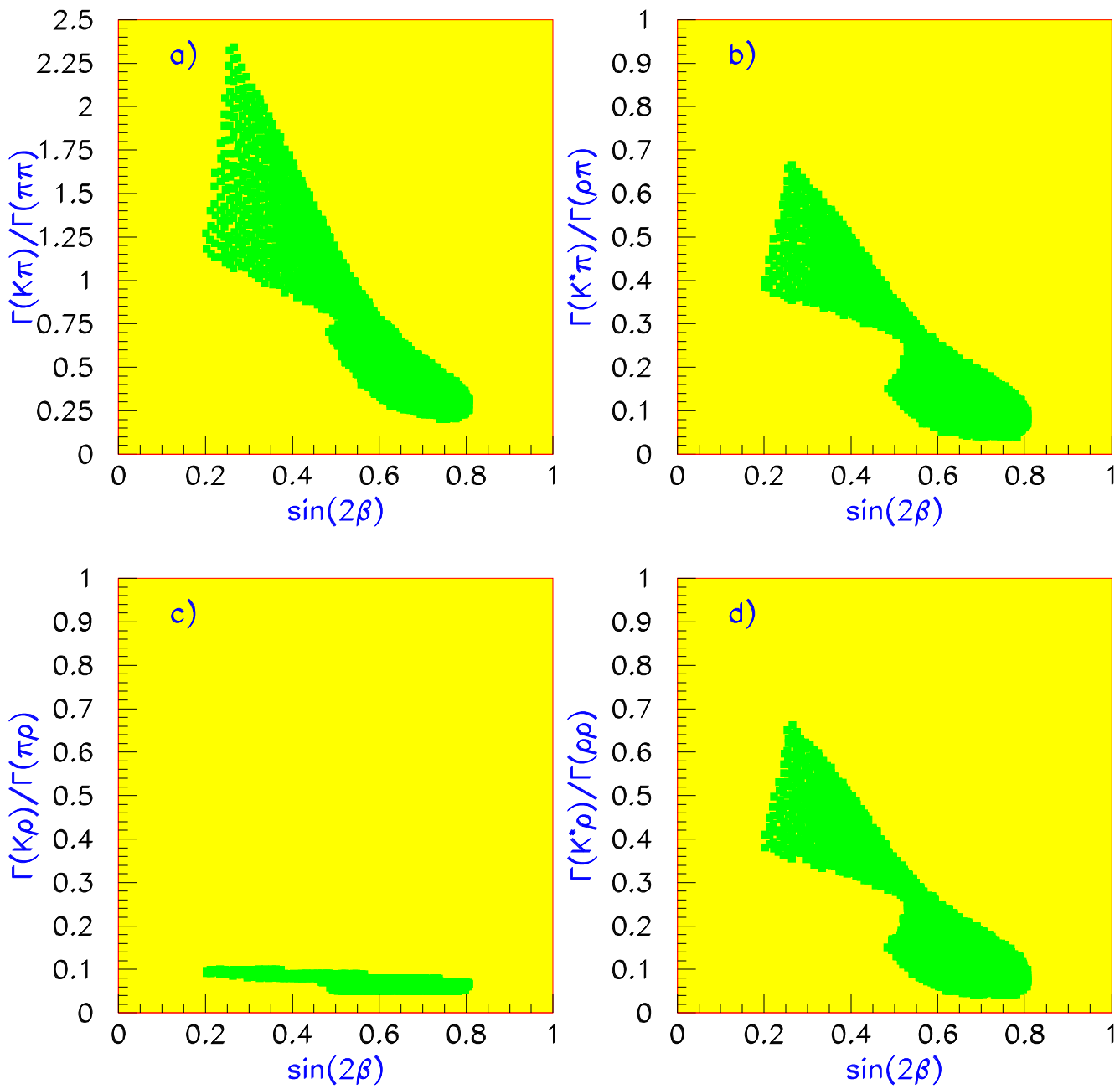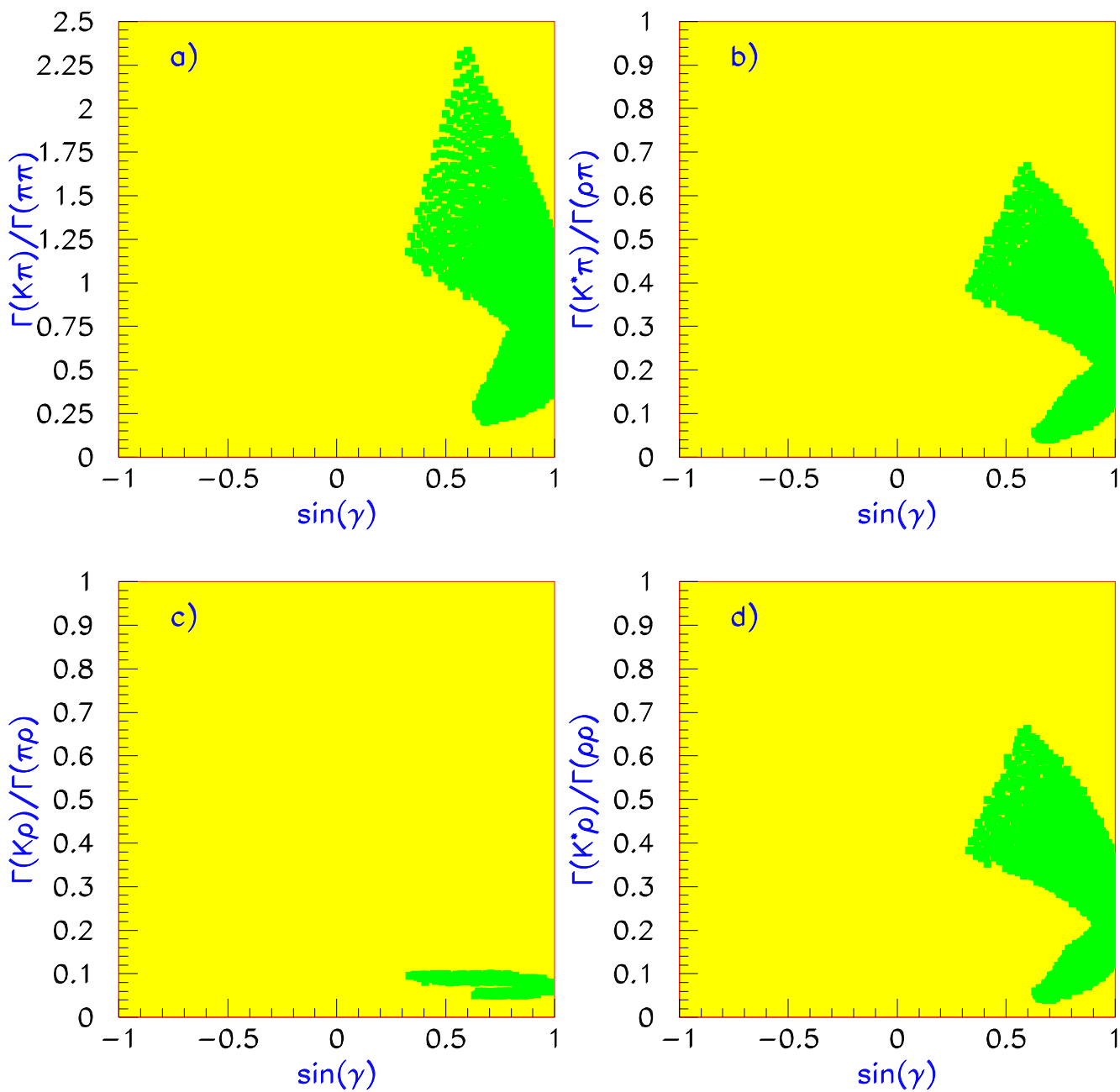
Figure 1

Figure 2

Figure 3

Figure 4

Figure 5